JJEE

Volume 7, Number 3, September 2021
Pages 289-303

Jordan Journal of Electrical Engineering

ISSN (Print): 2409-9600, ISSN (Online): 2409-9619

Centralized Robust Multi-Sensor Chandrasekhar-Type Recursive Least-Squares Wiener Filter in Linear Discrete-Time Stochastic Systems with Uncertain Parameters

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Received: March 08, 2021 *Revised: March 27, 2021* *Accepted: April 02, 2021*

***Abstract*—** In the centralized robust multi-sensor recursive least-square (RLS) Wiener filtering algorithm, the number of recursive equations increases compared to that of the centralized multi-sensor RLS Wiener filter in linear discrete-time stationary stochastic systems with uncertain parameters. Due to the increase in the number of recursive Riccati-type algebraic equations, the accumulation of round-off errors is not negligible. The round-off errors cause unstable numerical characteristics of the filter, especially for the small variance of the observation noise. To reduce the round-off errors - as the first attempt in the research field of centralized robust multi-sensor estimation - this paper designs the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter, which updates the filter gains recursively. To verify the effectiveness of the proposed filter, a numerical simulation example is demonstrated and its estimation accuracy is compared with the centralized robust multi-sensor RLS Wiener filter and the centralized multi-sensor RLS-Wiener filter. The obtained results show that the proposed filter exhibits better stability.

***Keywords*—** Chandrasekhar-type centralized robust RLS Wiener filter; Multi-sensor information fusion; Base station; Autoregressive model; Uncertain stochastic systems.

1. INTRODUCTION

Recently, robust recursive least-squares (RLS) Wiener estimators for linear discrete-time stationary stochastic systems with uncertain parameters have been proposed [1-5]. The estimators do not use the information about the uncertain parameters in the system and observation matrices at all, and they are also applicable to the system with randomly delayed observations [4]. Nakamori proposed a robust extended RLS Wiener filter and fixed-point smoother in discrete-time stationary stochastic systems [6]. In [7], distributed fusion estimation algorithms for multi-sensor networked systems are reviewed. In [8], the problem of distributed weighted robust Kalman filter fusion is studied in a class of uncertain systems with correlated noises. It is assumed that the system matrix includes zero-mean multiplicative noise with unity variance. In [9-11], the weighted fusion robust time-varying Kalman predictor, filter, and smoother are designed for multi-sensor time-varying systems with uncertainties of noise variances under the condition that the upper bounds of noise variances are given. In [12], the optimal centralized fusion filter, predictor, and smoother are presented in the linear minimum variance sense. It is assumed that correlated multiplicative noises exist in the state and observation matrices and their covariance data are known. In [13], robust centralized fusion and weighted measurement fusion Kalman estimators are designed for uncertain multi-sensor systems. The system matrix contains multiplicative uncertainty with known variance. The observation equation contains mutually uncorrelated scalar Bernoulli random variables with known probability. Upper bounds of the input and observation noise variances are known. In [14], centralized fusion of unscented Kalman filter

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Article's DOI: 10.5455/jjee.204-1615182000

is proposed based on the Huber robust method for nonlinear moving target tracking. Recently, Nakamori introduced the centralized robust multi-sensor RLS Wiener filter and fixed-point smoother in linear discrete-time stochastic systems with uncertain parameters [15]. In the case of the centralized robust multi-sensor RLS Wiener estimators in [8-13], the uncertain information about the state and observation equations is not required.

In the numerical simulation example - presented in this paper in section 6 - the mean-square values (MSVs) of the filtering errors by the centralized robust multi-sensor RLS Wiener filter are large for the white Gaussian observation noises, $\mathrm{N}(0,0.2^2)$ and $\mathrm{N}(0,0.3^2)$, respectively. The main reason the centralized robust RLS Wiener filter becomes unstable for multiple sensors is that $S_0(k)$, updated by the algebraic Riccati equations, becomes indefinite (e. g. $S_0(k) < 0$). This instability could be caused by the accumulation of the round-off errors in computing the Riccati equations for the symmetric matrix $S_0(k)$. $S_0(k)$ containing $N \cdot M \cdot m \times N \cdot M \cdot m$ equations (see section 4). This number is proportional to the square of the number of multi-sensor measurement points m.

In order to reduce the accumulation of the round-off errors, this paper proposes the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter in Theorem 1. The filter is considered the first attempt in the research field of centralized robust multi-sensor estimation. In Theorem 1, the filter gains $h(k,k)$ for $\hat{x}(k,k)$ and $\bar{h}(k,k)$ for $\hat{\hat{x}}(k,k)$ are recursively updated by Eqs. (14) and (16), respectively without including algebraic Riccati equations. Moreover, the numerical simulation example shows that the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1 is stable for the relatively small variance of the white Gaussian observation noise compared to the centralized robust multi-sensor RLS Wiener filter [15]. The MSVs of the filtering errors by the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type are smaller than those of the filtering errors by the centralized robust multi-sensor RLS Wiener filter and the centralized multi-sensor RLS Wiener filter for the white Gaussian observation noises $\mathrm{N}(0,0.2^2)$ and $\mathrm{N}(0,0.3^2)$, respectively. For the relatively small variance of the observation noise, the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar-type is effective from the viewpoint of estimation accuracy against the instability of the centralized robust multi-sensor RLS Wiener filter.

The rest of this paper is organized as following: section 2 introduces the problem of robust estimation for centralized multi-sensor information fusion in wide-sense stationary stochastic systems. In section 3, Theorem 1 proposes the centralized robust multi-sensor RLS Wiener filtering algorithm of Chandrasekhar type; Theorem 2 introduces the centralized robust multi-sensor RLS Wiener filtering algorithm [15] and Theorem 3 introduces the centralized multi-sensor RLS Wiener filtering algorithm [16]. Regarding the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type, section 4 proposes the recursive algorithm for the filtering error variance function of the state and shows the existence of its filtering estimate. Section 5 demonstrates a numerical simulation example.

2. DEGRADED SIGNALS IN LINEAR MULTI-SENSOR WIDE-SENSE STATIONARY STOCHASTIC SYSTEMS

In linear discrete-time wide-sense stationary stochastic systems, let the multi-sensor signals, $z_i(k),\ i=1,2,\cdots m,$ be observed at the local stations with additional observation noises $v_i(k)$ for the state equation for $x(k)$, given by Eq. (1).

$$\begin{aligned}
&y_i(k)=z_i(k)+v_i(k), z_i(k)=H_i x(k), i=1,2,\cdots m,\\
&y(k)=z(k)+v(k),\\
&y(k)=\begin{bmatrix} y_1(k)\\ y_2(k)\\ \vdots\\ y_m(k)\end{bmatrix}, z(k)=Hx(k)=\begin{bmatrix} z_1(k)\\ z_2(k)\\ \vdots\\ z_m(k)\end{bmatrix}, H=\begin{bmatrix} H_1\\ H_2\\ \vdots\\ H_m\end{bmatrix}, v(k)=\begin{bmatrix} v_1(k)\\ v_2(k)\\ \vdots\\ v_m(k)\end{bmatrix},\\
&E\left[v_i(k)v_i^T(s)\right]=R_i\delta_K(k-s), E\left[v_i(k)v_j^T(s)\right]=0, i\neq j, i,j=1,2,\cdots m,\\
&E[v(k)v^T(s)]=R\delta_K(k-s), R=\begin{bmatrix} R_1 & 0 & \cdots & 0 & 0\\ 0 & R_2 & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & R_{m-1} & 0\\ 0 & 0 & \cdots & 0 & R_m\end{bmatrix},\\
&x(k+1)=\Phi x(k)+\Gamma w(k), E[w(k)w^T(s)]=Q\delta_K(k-s)
\end{aligned} \tag{1}$$

Here, $z(k)$: $m\cdot M\times 1$ signal vector with components of m multi-sensor signals $z_i(k)$; $i=1,2,\cdots m$; H_i : $M\times \mathrm{n}$ observation matrix; $x(k)$: $n\times 1$ state vector to be estimated; $v_i(k)$: zero-mean white observation noise with variance R_i ; Φ: state transition matrix; $w(k)$: white input noise with variance Q; Γ: $n\times l$ input matrix. The notations $y(k), z(k)$ and $v(k)$ stand for the stacked vectors of $y_i(k)$, $z_i(k)$ and $v_i(k)$ vectors, $i=1,2,\cdots m$, respectively. The auto-covariance function of $v(k)$ is given in Eq. (1). Let the processes of the signals $z_i(k)$ and the observation noises $v_i(k)$ be independent of each other. Now, let the degraded multi-sensor observations $\breve{y}_i(k)$, $\mathrm{i}=1,2,\ldots,\mathrm{m}$, be generated by the state-space model with uncertain quantities $\Delta\Phi(k)$ in the system matrix and $\Delta \mathrm{H}_i(\mathrm{k})$ in the observation matrices. Let $\breve{y}_i(k)$, $i=1,2,\cdots m,$ be given as the sum of the degraded signal $\breve{z}_i(k)$ and the white observation noise $v_i(k)$ at the ith sensor.

$$\begin{aligned}
&\breve{y}_i(k)=\breve{z}_i(k)+v_i(k), \breve{z}_i(k)=\breve{H}_i(k)\breve{x}(k),\\
&\breve{x}(k+1)=\breve{\Phi}(k)\breve{x}(k)+\Gamma w(k),\\
&\breve{\Phi}(k)=\Phi+\Delta\Phi(k), \breve{H}_i(k)=H_i+\Delta \mathrm{H}_i(k), \mathrm{i}=1,\ldots,\mathrm{m}
\end{aligned} \tag{2}$$

Let $\breve{y}(k)$ and $\breve{z}(k)$ be the stacked vectors of $\breve{y}_i(k)$ and $\breve{z}_i(k)$, $i=1,2,\cdots m$. Then the observation equations - represented in Eq. (2) - are expressed with the stacked vectors as:

$$\begin{aligned}
&\breve{y}(k)=\breve{z}(k)+v(k),\\
&\breve{y}(k)=\begin{bmatrix} \breve{y}_1(k)\\ \breve{y}_2(k)\\ \vdots\\ \breve{y}_m(k)\end{bmatrix}, \ \breve{z}(k)=\begin{bmatrix} \breve{z}_1(k)\\ \breve{z}_2(k)\\ \vdots\\ \breve{z}_m(k)\end{bmatrix}.
\end{aligned} \tag{3}$$

Suppose the process of the degraded multi-sensor signal $\breve{z}(k)$ is fitted to the multivariate autoregressive (AR) model of finite order N.

$$\begin{aligned}
&\breve{z}(k)=-\overleftrightarrow{a}_1\breve{z}(k-1)-\overleftrightarrow{a}_2\breve{z}(k-2)\cdots-\overleftrightarrow{a}_N\breve{z}(k-N)+\overleftrightarrow{e}(k),\\
&E[\overleftrightarrow{e}(k)\overleftrightarrow{e}^T(s)]=\overleftrightarrow{Q}\delta_K(k-s)
\end{aligned} \tag{4}$$

Let us introduce, among the multi-sensor observations for $m\geq 2$, the multi-sensor state $\overleftrightarrow{x}(k)$ with components $\breve{z}_1(k), \breve{z}_2(k), \breve{z}_3(k), \ldots, \breve{z}_m(k), \breve{z}_1(k+1), \breve{z}_2(k+1), \ldots, \breve{z}_m(k+1),$

$\cdots$, $\check{z}_1(k+N-2)$, $\check{z}_2(k+N-2)$, $\cdots$, $\check{z}_m(k+N-2)$, $\check{z}_1(k+N-1)$, $\check{z}_2(k+N-1)$, $\cdots$, $\check{z}_m(k+N-1)$. With the observation matrix $\overleftrightarrow{H}$ and the state $\overleftrightarrow{x}(k)$, we express the degraded multi-sensor signal $\check{z}(k)$ as:

$$\check{z}(k) = \overleftrightarrow{H}\overleftrightarrow{x}(k), \overleftrightarrow{H} = [I_{M\cdot m\times M\cdot m} \quad 0 \quad \cdots \quad 0 \quad 0],$$

$$\overleftrightarrow{x}(k) = \begin{bmatrix} \check{z}(k) \\ \check{z}(k+1) \\ \vdots \\ \check{z}(k+N-2) \\ \check{z}(k+N-1) \end{bmatrix} = \begin{bmatrix} \check{z}_1(k) \\ \check{z}_2(k) \\ \vdots \\ \check{z}_m(k) \\ \check{z}_1(k+1) \\ \check{z}_2(k+1) \\ \vdots \\ \check{z}_m(k+1) \\ \vdots \\ \check{z}_1(k+N-2) \\ \check{z}_2(k+N-2) \\ \vdots \\ \check{z}_m(k+N-2) \\ \check{z}_1(k+N-1) \\ \check{z}_2(k+N-1) \\ \vdots \\ \check{z}_m(k+N-1) \end{bmatrix}. \tag{5}$$

From Eqs. (4) and (5), we see that the state equation for $\overleftrightarrow{x}(k)$ satisfies:

$$\overleftrightarrow{x}(k+1) = \overleftrightarrow{\Phi}\overleftrightarrow{x}(k) + \overleftrightarrow{\Gamma}\overleftrightarrow{w}(k),$$

$$\overleftrightarrow{\Phi} = \begin{bmatrix} 0 & I_{M\cdot m\times M\cdot m} & 0 & \cdots & 0 \\ 0 & 0 & I_{M\cdot m\times M\cdot m} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I_{M\cdot m\times M\cdot m} \\ -\overleftrightarrow{a}_N & -\overleftrightarrow{a}_{N-1} & -\overleftrightarrow{a}_{N-2} & \cdots & -\overleftrightarrow{a}_1 \end{bmatrix}, \overleftrightarrow{\Gamma} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ I_{M\cdot m\times M\cdot m} \end{bmatrix} \tag{6}$$

$$E[\overleftrightarrow{w}(k)\overleftrightarrow{w}^T(s)] = \overleftrightarrow{Q}\delta_K(k-s), \overleftrightarrow{w}(k) = \overleftrightarrow{e}(k+N),$$

with the system matrix $\overleftrightarrow{\Phi}$ in the controllable canonical form. By using the auto-covariance function of the degraded multi-sensor signal $\check{z}(k)$, $\check{K}(k,s) = E[\check{z}(k)\check{z}^T(s)] = \check{K}(i)$, $i = k-s$, $0 \le i \le N$, the AR parameters, $\overleftrightarrow{a}_i$, $1 \le i \le N$, are calculated by the Yule-Walker equations [15].

$$\overleftrightarrow{K}(k,k) \begin{bmatrix} \overleftrightarrow{a}_1^T \\ \overleftrightarrow{a}_2^T \\ \vdots \\ \overleftrightarrow{a}_{N-1}^T \\ \overleftrightarrow{a}_N^T \end{bmatrix} = - \begin{bmatrix} \check{K}^T(1) \\ \check{K}^T(2) \\ \vdots \\ \check{K}^T(N-1) \\ \check{K}^T(N) \end{bmatrix} \tag{7}$$

Here, the auto-variance function $\overleftrightarrow{K}(k,k)$ of the multi-sensor state $\overleftrightarrow{x}(k)$ is given by:

$$\overleftrightarrow{K}(k,k) = E[\overleftrightarrow{x}(k)\overleftrightarrow{x}^T(k)] = \begin{bmatrix} \check{K}(0) & \check{K}(1) & \cdots \check{K}(N-2) & \check{K}(N-1) \\ \check{K}^T(1) & \check{K}(0) & \cdots \check{K}(N-3) & \check{K}(N-2) \\ \vdots & \vdots & \ddots \quad \vdots & \vdots \\ \check{K}^T(N-2) & \check{K}^T(N-3) & \cdots \quad \check{K}(0) & \check{K}(1) \\ \check{K}^T(N-1) & \check{K}^T(N-2) & \cdots \quad \check{K}^T(1) & \check{K}(0) \end{bmatrix}. \tag{8}$$

Also, the cross-covariance function $K_{x\overleftrightarrow{x}}(k,s) = E[x(k)\overleftrightarrow{x}^T(s)]$ of the state $x(k)$ with the state $\overleftrightarrow{x}(s)$ is given by:

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$$K_{x\overleftrightarrow{x}}(k,s) = \alpha(k)\beta^T(s), 0 \le s \le k,$$
$$\alpha(k) = \Phi^k, \beta^T(s) = \Phi^{-s}K_{x\overleftrightarrow{x}}(s,s). \tag{9}$$

Assume that the filtering estimate $\hat{x}(k,k)$ of $x(k)$ is given by:

$$\hat{x}(k,k) = \sum_{i=1}^{k} h(k,i)\, \breve{y}(i) \tag{10}$$

From the lemma of orthogonal projection [3], it follows that the optimal impulse response function $h(k,s)$ satisfies:

$$h(k,s)R = K_{x\overleftrightarrow{x}}(k,s)\overleftrightarrow{H}^T - \sum_{i=1}^{k} h(k,i)\overleftrightarrow{H}\,\overleftrightarrow{K}(i,k)\overleftrightarrow{H}^T \tag{11}$$

In section 3, starting from Eq. (11), we obtain the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type.

3. CENTRALIZED ROBUST MULTI-SENSOR RLS WIENER FILTERING ALGORITHM OF CHANDRASEKHAR TYPE

Based on the preliminary assumptions on the centralized robust multi-sensor estimation problem in section 2, Theorem 1 introduces the centralized robust multi-sensor RLS Wiener filtering algorithm of Chandrasekhar type for estimating the signal $z(k)$ and the state $x(k)$ in linear wide-sense stationary stochastic systems with uncertain parameters in the system and observation matrices.

Theorem 1: Let the state-space model for state $x(k)$ be given by Eq. (1). Let the state-space model with uncertain quantities $\Delta\Phi(k)$ and $\Delta H_i(k)$, $i = 1, \dots, m$, be given by Eq. (2); the process of the degraded multi-sensor signal $\breve{z}(k)$ be fitted to the AR model of the order N; the variance $\overleftrightarrow{K}(k,k)$ of the multi-sensor state $\overleftrightarrow{x}(k)$ and the cross-variance $K_{x\overleftrightarrow{x}}(k,k)$ of the state $x(k)$ with the multi-sensor state $\overleftrightarrow{x}(k)$ be given by Eqs. (8) and (9), respectively; the variance of the multi-sensor white observation noise $v(k)$ be R. Then, the centralized robust multi-sensor RLS Wiener filtering algorithm of Chandrasekhar type for the signal $z(k)$ and the state $x(k)$ consists of Eqs. (12) to (20) in linear wide-sense stationary stochastic systems with the uncertain parameters in the system and observation matrices.

Filtering estimate of signal $z(k)$: $\hat{z}(k,k)$

$$\hat{z}(k,k) = H\hat{x}(k,k) \tag{12}$$

Filtering estimate of the state $x(k)$, i. e. $\hat{x}(k,k)$

$$\hat{x}(k,k) = \Phi\hat{x}(k-1,k-1) + h(k,k)(\breve{y}(k) - \overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(k-1,k-1)),$$
$$\hat{x}(0,0) = 0 \tag{13}$$

Filter gain for $\hat{x}(k,k)$ in Eq. (13), i. e. $h(k,k)$

$$h(k,k) = [h(k-1,k-1) - \Phi h(k-1,1)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1)]$$
$$\times \{I - \overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1)\}^{-1} \tag{14}$$

Filtering estimate of $\overleftrightarrow{x}(k)$, i. e. $\hat{\overleftrightarrow{x}}(k,k)$

$$\hat{\overleftrightarrow{x}}(k,k) = \overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(k-1,k-1) + \bar{h}(k,k)(\breve{y}(k) - \overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(k-1,k-1)),$$
$$\hat{\overleftrightarrow{x}}(0,0) = 0 \tag{15}$$

Filter gain for $\hat{\overleftrightarrow{x}}(k,k)$ in Eq. (15), i. e. $\bar{h}(k,k)$

$$\bar{h}(k,k) = [\bar{h}(k-1,k-1) - \overleftrightarrow{\Phi}\bar{h}(k-1,1)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1)]$$
$$\times \{I - \overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1)\}^{-1} \tag{16}$$

Update equation of $h(k,1)$ from $h(k-1,1)$.

$$h(k,1) = \Phi h(k-1,1) - h(k,k)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1) \tag{17}$$

Update equation of $\bar{h}(k,1)$ from $\bar{h}(k-1,1)$.

$$\bar{h}(k,1)=\overleftrightarrow{\Phi}\bar{h}(k-1,1)-\bar{h}(k,k)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1) \tag{18}$$

The Initial value of Eqs. (14) and (17), i. e. $h(1,1)$

$$h(1,1)=K_{x\overleftrightarrow{x}}(1,1)\overleftrightarrow{H}^{T}(R+\overleftrightarrow{H}\overleftrightarrow{K}(k,k)\overleftrightarrow{H}^{T})^{-1} \tag{19}$$

The Initial value of Eqs. (16) and (18), i. e. $h(1,1)$

$$\bar{h}(1,1)=\overleftrightarrow{K}(1,1)\overleftrightarrow{H}^{T}(R+\overleftrightarrow{H}\overleftrightarrow{K}(k,k)\overleftrightarrow{H}^{T})^{-1} \tag{20}$$

The Chandrasekhar-type centralized robust multi-sensor RLS filter of Theorem 1 is derived by applying the mathematical procedure developed in Eqs. (1 - 11) by Nakamori et al. [17]. The proof of Theorem 1 is deferred to the Appendix.

For the stability of the Chandrasekhar type centralized robust multi-sensor RLS Wiener filter of Theorem 1, the following conditions are required: 1) $R+\overleftrightarrow{H}\overleftrightarrow{K}(k,k)\overleftrightarrow{H}^{T}$ and $I-\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1)$ are positive definite matrices; 2) The system matrix Φ is stable; 3) The matrix $\overleftrightarrow{\Phi}-\bar{h}(k,k)\overleftrightarrow{H}\overleftrightarrow{\Phi}$ is stable. Conditions 2 and 3 indicate that all the eigenvalues of the matrices Φ and $\overleftrightarrow{\Phi}-\bar{h}(k,k)\overleftrightarrow{H}\overleftrightarrow{\Phi}$ lie inside the unit circle.

Theorem 2: Let us apply the same assumptions of section 2 to the state-space model for the state $x(k)$, the uncertain quantities $\Delta\Phi(k)$ and $\Delta H_i(k)$, $\mathrm{i}=1,\ldots,\mathrm{m}$, the degraded multi-sensor signal $\breve{z}(k)$ fitted to the AR model of order N, the variance $\overleftrightarrow{K}(k,k)$ of the multi-sensor state $\overleftrightarrow{x}(k)$, the cross-variance $K_{x\overleftrightarrow{x}}(k,k)$ of the state $x(k)$ with $\overleftrightarrow{x}(k)$ and the variance of the white multi-sensor observation noise $v(k)$. Then the centralized multi-sensor robust RLS Wiener filtering algorithm for the signal $z(k)$ and the state $x(k)$ consists of Eqs. (21) to (27) in linear wide-sense stationary stochastic uncertain systems [15].

Filtering estimate of signal $z(k)$, i. e. $\hat{z}(k,k)$:

$$\hat{z}(k,k)=H\hat{x}(k,k) \tag{21}$$

Filtering estimate of the state $x(k)$, i. e. $\hat{x}(k,k)$:

$$\hat{x}(k,k)=\Phi\hat{x}(k-1,k-1)+G(k)(\breve{y}(k)-\overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(k-1,k-1)),\quad \hat{x}(0,0)=0 \tag{22}$$

Filter gain for $\hat{x}(k,k)$ in Eq. (22), i. e. $G(k)$:

$$\begin{aligned}G(k)&=[K_{x\breve{z}}(k,k)-\Phi S(k-1)\overleftrightarrow{\Phi}^{T}\overleftrightarrow{H}^{T}]\\&\times\{R+\overleftrightarrow{H}[\overleftrightarrow{K}(k,k)-\overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^{T}]\overleftrightarrow{H}^{T}\}^{-1},\\K_{x\breve{z}}(k,k)&=K_{x\overleftrightarrow{x}}(k,k)\overleftrightarrow{H}^{T}\end{aligned} \tag{23}$$

Filtering estimate of $\overleftrightarrow{x}(k)$, i. e. $\hat{\overleftrightarrow{x}}(k,k)$:

$$\hat{\overleftrightarrow{x}}(k,k)=\overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(k-1,k-1)+g(k)(\breve{y}(k)-\overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(k-1,k-1)),\quad \hat{\overleftrightarrow{x}}(0,0)=0 \tag{24}$$

Filter gain for $\hat{\overleftrightarrow{x}}(k,k)$ in Eq. (24), i.e. $g(k)$:

$$\begin{aligned}g(k)&=[\overleftrightarrow{K}(k,k)\overleftrightarrow{H}^{T}-\overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^{T}\overleftrightarrow{H}^{T}]\\&\times\{R+\overleftrightarrow{H}[\overleftrightarrow{K}(k,k)-\overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^{T}]\overleftrightarrow{H}^{T}\}^{-1}\end{aligned} \tag{25}$$

Auto-variance function of $\hat{\overleftrightarrow{x}}(k,k)$:

$$S_0(k)=\overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^{T}+g(k)\overleftrightarrow{H}[\overleftrightarrow{K}(k,k)-\overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^{T}],\quad S_0(0)=0 \tag{26}$$

Cross-variance function of $\hat{x}(k,k)$ with $\hat{\overleftrightarrow{x}}(k,k)$:

$$S(k)=\Phi S(k-1)\overleftrightarrow{\Phi}^{T}+G(k)\overleftrightarrow{H}[\overleftrightarrow{K}(k,k)-\overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^{T}],\quad S(0)=0 \tag{27}$$

Next, Theorem 3 shows the centralized multi-sensor RLS Wiener filtering algorithm.

Theorem 3: Suppose the state-space model for state $x(k)$ is given by Eq. (1); then, the centralized multi-sensor RLS Wiener filtering algorithm consists of Eqs. (28) to (31). The centralized multi-sensor RLS Wiener filtering algorithm requires the information of the system matrix Φ, the observation matrix H, the auto-variance function $K_x(k,k)$ of $x(k)$, and the degraded multi-sensor observed value $\breve{y}(k)$ in linear discrete-time wide-sense stationary stochastic systems [15].

Filtering estimate of signal $z(k)$, i.e. $\hat{z}(k,k)$:

$$\hat{z}(k,k) = H\hat{x}(k,k) \tag{28}$$

Filtering estimate of the state $x(k)$, i. e. $\hat{x}(k,k)$:

$$\hat{x}(k,k) = \Phi\hat{x}(k-1,k-1) + G_x(k)(\breve{y}(k) - H\Phi\hat{x}(k-1,k-1)), \quad \hat{x}(0,0) = 0 \tag{29}$$

Filter gain for $\hat{x}(k,k)$ in Eq. (26), i. e. $G_x(k)$:

$$G_x(k) = [(K_x(k,k) - \Phi S_x(k-1)\Phi^T)H^T] \times \{R + H[K_x(k,k) - \Phi S_x(L-1)\Phi^T]H^T\}^{-1} \tag{30}$$

The variance of the filtering estimate $\hat{x}(k,k)$, i. e. $S_x(k)$:

$$S_x(k) = \Phi S_x(k-1)\Phi^T + G_x(k)H[K_x(k,k) - \Phi S_x(k-1)\Phi^T], \quad S_x(0) = 0 \tag{31}$$

In section 4, we present the recursive algorithm for the filtering error variance function of the state $x(k)$ for the centralized robust multi-sensor RLS Wiener filtering algorithm of Chandrasekhar type and show the existence of the state.

4. COMPARISON OF THE NUMBER OF RECURSIVE EQUATIONS IN THEOREM 1, THEOREM 2, AND THEOREM 3

The numbers of recursive equations contained in Theorem 1, Theorem 2, and Theorem 3 are specified as follows.

Theorem 1: Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1: $N \cdot M \cdot m$ for $\hat{x}(k,k)$; $N \cdot M \cdot m$ for $h(k,k)$; $M^2 \cdot N \cdot m^2$ for $\hat{\hat{x}}(k,k)$; $M^2 \cdot N \cdot m^2$ for $\bar{h}(k,k)$; $N \cdot M \cdot m$ for $h(k,1)$ and $M^2 \cdot N \cdot m^2$ for $\bar{h}(k,1)$. The total number equals $3N \cdot M \cdot m$+$3M^2 \cdot N \cdot m^2$

Theorem 2: Centralized robust multi-sensor RLS Wiener filter of Theorem 2: $M \cdot N \cdot m$ for $\hat{x}(k,k)$; $M^2 \cdot N \cdot m^2$ for $\hat{\hat{x}}(k,k)$; $M^2 \cdot N^2 \cdot m^2$ for $S_0(k)$and $M \cdot N^2 \cdot m$ for $S(k)$. The total number is: $M^2 \cdot N^2 \cdot m^2 + M^2 \cdot N \cdot m^2 + M \cdot N^2 \cdot m + M \cdot N \cdot m$

Theorem 3: Centralized multi-sensor RLS Wiener filter of Theorem 3: $M \cdot N \cdot m$ for $\hat{x}(k,k)$ and N^2 for $S_x(k)$. The total number is: $N^2 + M \cdot N \cdot m$

The main reason the centralized robust RLS Wiener filter becomes unstable for multiple sensors is that $S_0(k)$, updated by the algebraic Riccati equations becomes indefinite, e. g. $S_0(k) < 0$ [18]. This instability could be caused by the accumulation of the round-off errors in the computation of the Riccati equations for the symmetric matrix $S_0(k)$. $S_0(k)$ containing $N \cdot M \cdot m \times N \cdot M \cdot m$ equations. In the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1, the filter gains $h(k,k)$ for $\hat{x}(k,k)$ and $\bar{h}(k,k)$ for $\hat{\hat{x}}(k,k)$ are recursively updated by Eqs. (14) and (16) without algebraic Riccati equations. As explained in the numerical simulation example in section 6, the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1 is stable for the relatively small variance of the white Gaussian observation noise compared to the centralized robust multi-sensor RLS Wiener filter [15].

5. FILTERING ERROR VARIANCE FUNCTION FOR STATE $x(k)$

This section presents the recursive algorithm for the filtering error variance function $P_{\tilde{x}}(k)$ of the state $x(k)$ in the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filtering algorithm of Theorem 1. From Eq. (13), the variance $P_{\hat{x}}(k,k)$ of the filtering estimate $\hat{x}(k,k)$ is given by:

$$\begin{aligned} P_{\hat{x}}(k) &= \mathrm{E}[\hat{x}(k,k)\hat{x}^T(k,k)] \\ &= \Phi \mathrm{E}[\hat{x}(k-1,k-1)\hat{x}^T(k-1,k-1)]\Phi^T + h(k,k)P_{\check{v}}(k)h^T(k,k) \\ &= \Phi P_{\hat{x}}(k-1)\Phi^T + h(k,k)P_{\check{v}}(k)h^T(k,k) \end{aligned} \tag{32}$$

$$P_{\check{v}}(k) = E[\check{v}(k)\check{v}(k)^T(k)], \check{v}(k) = \check{y}(k) - \overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\hat{x}}(k-1,k-1).$$

$\check{v}(k)$ represents the innovation process. The variance of the innovation process $P_{\check{v}}(k)$ is expressed as:

$$\begin{aligned} P_{\check{v}}(k) &= \mathrm{E}\left[(\check{y}(k) - \overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\hat{x}}(k-1,k-1))(\check{y}(k) - \overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\hat{x}}(k-1,k-1))^T\right] \\ &= \overleftrightarrow{H}\overleftrightarrow{K}(k,k)\overleftrightarrow{H}^T + R - \overleftrightarrow{H}\overleftrightarrow{\Phi}P_{\hat{\hat{x}}}(k-1)\overleftrightarrow{\Phi}^T\overleftrightarrow{H}^T, \end{aligned} \tag{33}$$

$$P_{\hat{\hat{x}}}(k) = E[\hat{\hat{x}}(k,k)\hat{\hat{x}}^T(k,k)].$$

Here, $P_{\hat{\hat{x}}}(k)$ represents the variance of the filtering estimate $\hat{\hat{x}}(k,k)$. From Eq. (15), we have an expression for $P_{\hat{\hat{x}}}(k)$ as follows.

$$P_{\hat{\hat{x}}}(k) = \overleftrightarrow{\Phi}P_{\hat{\hat{x}}}(k-1)\overleftrightarrow{\Phi} + \bar{h}(k,k)P_{\check{v}}(k)\bar{h}^T(k,k). \tag{34}$$

Hence, we get Eqs. (35) and (36):

$$P_{\hat{x}}(k) = \Phi P_{\hat{x}}(k-1)\Phi^T + h(k,k)[\overleftrightarrow{H}\overleftrightarrow{K}(k,k)\overleftrightarrow{H}^T + R - \overleftrightarrow{H}\overleftrightarrow{\Phi}P_{\hat{\hat{x}}}(k-1)\overleftrightarrow{\Phi}^T\overleftrightarrow{H}^T]h^T(k,k) \tag{35}$$

$$P_{\hat{x}}(0) = 0$$

$$P_{\hat{\hat{x}}}(k) = \overleftrightarrow{\Phi}P_{\hat{\hat{x}}}(k-1)\overleftrightarrow{\Phi} + \bar{h}(k,k)\left[\overleftrightarrow{H}\overleftrightarrow{K}(k,k)\overleftrightarrow{H}^T + R - \overleftrightarrow{H}\overleftrightarrow{\Phi}P_{\hat{\hat{x}}}(k-1)\overleftrightarrow{\Phi}^T\overleftrightarrow{H}^T\right]\bar{h}^T(k,k), \tag{36}$$

$$P_{\hat{\hat{x}}}(0) = 0.$$

The filtering error variance function $P_{\tilde{x}}(k)$ of $x(k)$ is given by:

$$P_{\tilde{x}}(k,k) = K_x(k,k) - P_{\hat{x}}(k,k), \tag{37}$$

where $K_x(k,k)$ represents the variance of the state $x(k)$. The recursive algorithm for the filtering error variance function $P_{\tilde{x}}(k)$ consists of Eqs. (14), (16-20), and (35-37).

The variance $P_{\tilde{x}}(k)$ of the filtering estimate $\hat{x}(k,k)$ of the state $x(k)$ is lower bounded by the zero matrix and upper bounded by the variance $K_x(k,k)$ of the state.

$$0 \le P_{\hat{x}}(k) \le K_x(k,k)$$

From this inequality, we see that the existence of the filtering estimate $\hat{x}(k,k)$ of the state is shown.

The following shows a numerical simulation example of the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1. Its estimation characteristics are compared with the centralized robust multi-sensor RLS Wiener filter [15] and the centralized multi-sensor RLS Wiener filter [16].

6. A NUMERICAL SIMULATION EXAMPLE

Suppose that the observation equations in the two-sensor information fusion network system and the state equation for $x(k)$ are described by:

$$
\begin{aligned}
&y_i(k) = z_i(k) + v_i(k), z_i(k) = H_i x(k), i = 1,2,\\
&y(k) = z(k) + v(k), z(k) = Hx(k), H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}, x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix},\\
&H_1 = [1 \quad -0.1], H_2 = [0.1 \quad 1],\\
&y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix}, z(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} = \begin{bmatrix} x_1(k) - 0.1x_2(k) \\ 0.1x_1(k) + x_2(k) \end{bmatrix}, v(k) = \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix},\\
&x(k+1) = \Phi x(k) + \Gamma w(k), \Phi = \begin{bmatrix} 0 & 1 \\ 0.8 & 0.1 \end{bmatrix}, \Gamma = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\\
&E[v(k)v(s)] = R\delta_K(k-s), R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}, R_1 = R_2,\\
&E[w(k)w(s)] = Q\delta_K(k-s), Q = 0.5^2.
\end{aligned} \tag{38}
$$

From $m = 2$, $M = 1$, and $N = 5$, the total numbers of the recursive equations in Theorems 1, 2, and 3 are 90, 180, and 35, respectively. The number of Riccati-type equations for $S_0(k)$ in Theorem 2 is 100 and that for $S_x(k)$ in Theorem 3 is 25. Suppose that the degraded observed value $\breve{y}(k)$ is generated by the observation Eq. (39). Here, the degraded observation $\breve{y}(k)$ has the two components $\breve{y}_1(k)$ and $\breve{y}_2(k)$. The degraded signal $\breve{z}(k)$ has the components $\breve{z}_1(k)$ and $\breve{z}_2(k)$.

$$
\begin{aligned}
&\breve{y}(k) = \breve{z}(k) + v(k), \breve{z}(k) = \breve{H}(k)\breve{x}(k), \breve{y}(k) = \begin{bmatrix} \breve{y}_1(k) \\ \breve{y}_2(k) \end{bmatrix}, \breve{z}(k) = \begin{bmatrix} \breve{z}_1(k) \\ \breve{z}_2(k) \end{bmatrix},\\
&\breve{H}(k) = \begin{bmatrix} \breve{H}_1(k) \\ \breve{H}_2(k) \end{bmatrix}, \breve{x}(k) = \begin{bmatrix} \breve{x}_1(k) \\ \breve{x}_2(k) \end{bmatrix}
\end{aligned} \tag{39}
$$

We assume that the state-space model contains uncertain quantities $\Delta H_i(k), i = 1,2$, and $\Delta\Phi(k)$ as shown in Eq. (40).

$$
\begin{aligned}
&\breve{y}_i(k) = \breve{z}_i(k) + v_i(k), \breve{z}_i(k) = \breve{H}_i(k)\breve{x}(k),\\
&\breve{x}(k+1) = \breve{\Phi}(k)\breve{x}(k) + \Gamma w(k), \qquad ,\\
&\breve{\Phi}(k) = \Phi + \Delta\Phi(k), \breve{H}_i(k) = H_i + \Delta H_i(k), i = 1,2,\\
&\Delta\Phi(k) = \begin{bmatrix} 0 & 0 \\ 0.2\zeta_1(k) & 0.1\zeta_2(k) \end{bmatrix},\\
&\Delta H_1(k) = [0.1\zeta_3(k) \quad 0], \Delta H_2(k) = [0.05\zeta_4(k) \quad 0]
\end{aligned} \tag{40}
$$

It should be noted that the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type, the centralized robust multi-sensor RLS Wiener filter and the multi-sensor RLS Wiener filter do not use the information of the uncertain quantities. Here, $\zeta_i(k)$, i = 1,2, … ,4, are mutually independent uniformly distributed random variables, which ranges between 0 and 1. Suppose that the degraded multi-sensor signal $\breve{z}(k)$ is approximated with the multivariate AR model in Eq. (4) of the order N = 5. In this case, the multi-sensor state $\overleftrightarrow{x}(k)$ of Eq. (5) has 10 vector components.

By substituting H, Φ, $\overleftrightarrow{H}$, $\overleftrightarrow{\Phi}$, $\overleftrightarrow{K}(L,L)$, $K_{x\overleftrightarrow{x}}(k,k)$ and R into Theorem 1, the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type recursively calculates the estimates of the states $x_1(k)$ and $x_2(k)$. Here, $\overleftrightarrow{K}(L,L)$ and $K_{x\overleftrightarrow{x}}(k,k)$ are computed with the data of $x(k)$ and $\overleftrightarrow{x}(k)$; 1 ≤ k ≤ 350. Fig. 1 illustrates the state x_1(k) and its filtering estimate $\hat{x}_1(k,k)$ vs. time k by the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1 for the white Gaussian observation noise $N(0,0.5^2)$. Fig. 2 illustrates the state x_2(k) and its filtering estimate $\hat{x}_2$(k, k) vs. time k by the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type for the white Gaussian observation noise $N(0,0.5^2)$. The centralized robust multi-sensor RLS Wiener filter uses the same information as the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type. The centralized multi-sensor RLS Wiener filter of Theorem 3 uses the information Φ, H and the auto-variance function of the state $x(k)$, $K_x(k,k)$. Here, the relationship $K_x(k,k) = K_x(0)$ holds from the wide-sense stationarity. $K_x(k,k)$ is computed iteratively by $K_x(k+1,k+1) = \Phi K_x(k,k)\Phi^T + \Gamma Q \Gamma^T$, with the initial value $K_x(k,k) = 0_{2\times 2}$, until $K_x(k,k)$ arrives at its stationary value.

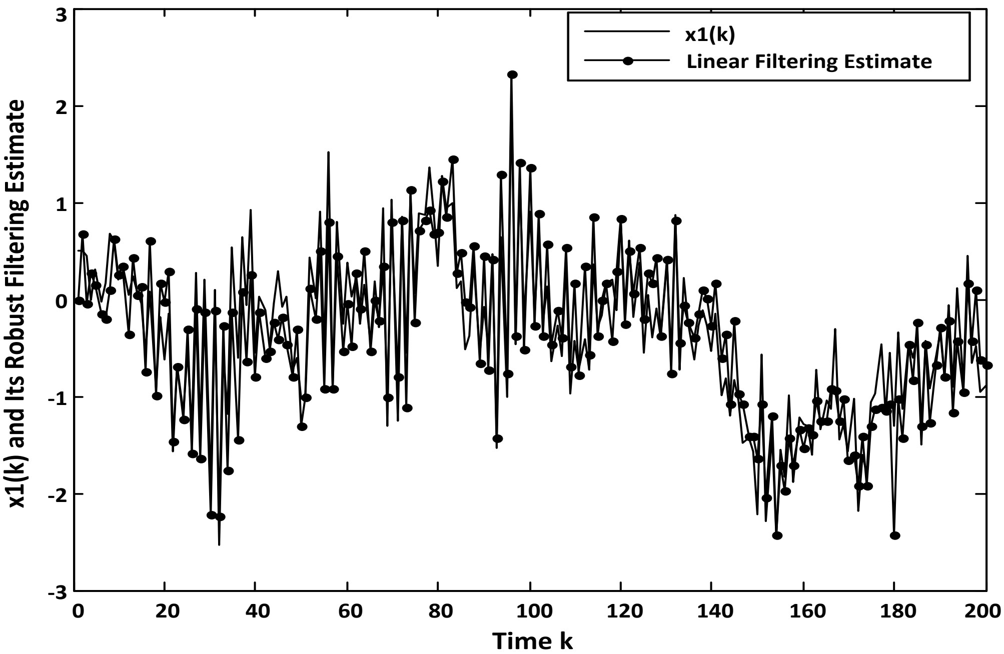

Fig. 1. Filtering estimate $\hat{x}_1(k,k)$ by the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1 vs. k for the white Gaussian observation noise $\mathrm{N}(0,0.5^2)$.

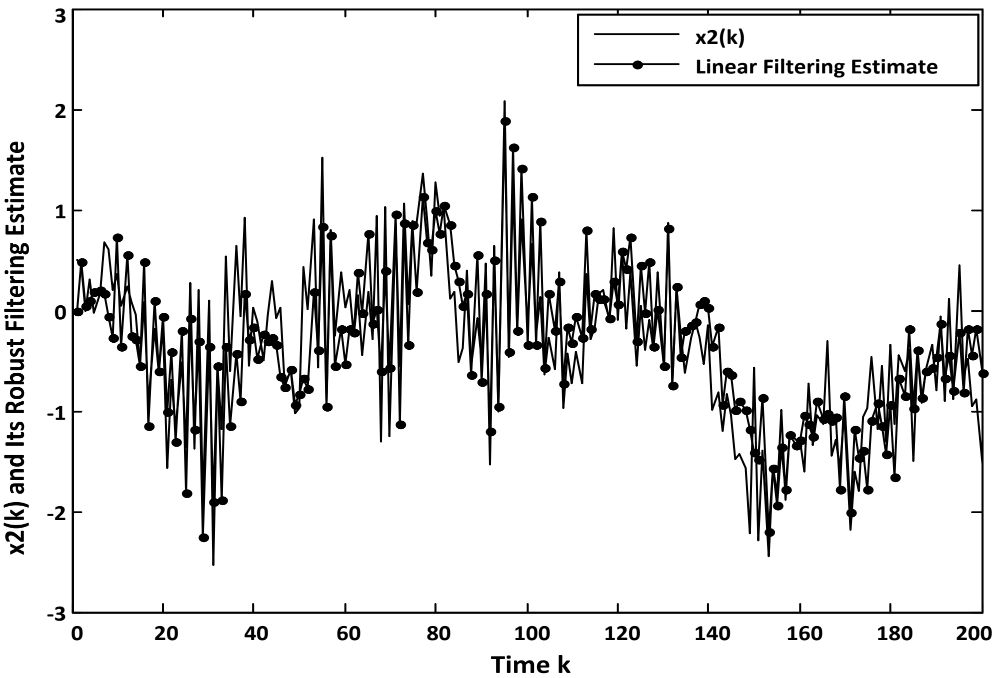

Fig. 2. Filtering estimate $\hat{x}_2(k,k)$ by the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1 vs. k for the white Gaussian observation noise $\mathrm{N}(0,0.5^2)$.

Table 1 compares the MSVs of the filtering errors of x2(k) between the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1, the centralized robust multi-sensor RLS Wiener filter [15] and the centralized multi-sensor RLS Wiener filter [16] for the white Gaussian observation noises $\mathrm{N}(0,0.2^2)$ and $\mathrm{N}(0,0.3^2)$. From Table 1 the MSVs of the filtering errors by the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type are smaller than those of the filtering errors by the centralized multi-sensor RLS Wiener filter [16] for the white Gaussian observation noises, $\mathrm{N}(0,0.2^2)$ and $\mathrm{N}(0,0.3^2)$, respectively. The MSVs of the filtering errors by the centralized robust multi-sensor RLS Wiener filter [15] are large for the white Gaussian observation noises, $\mathrm{N}(0,0.2^2)$ and $\mathrm{N}(0,0.3^2)$, respectively. These results show that the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type is effective in estimation accuracy, for the relatively small variance of the observation noise, against the instability of the centralized robust multi-sensor RLS Wiener filter.

Table 1. MSVs of filtering errors of x2(k) for different types of centralized multi-sensor RLS Wiener filters.

Observation noise	MSVs of filtering errors		
	Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1	centralized robust multi-sensor RLS Wiener filter [15]	centralized multi-sensor RLS Wiener filter [16]
$\mathrm{N}(0,0.2^2)$	0.0937	9.4690e+005	0.2441
$\mathrm{N}(0,0.3^2)$	0.1171	2.2048e+005	0.2170

Fig. 3 shows the MSVs of the filtering errors of $x_1(k)$ by the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1, the centralized robust multi-sensor RLS Wiener filter [15] and the centralized multi-sensor RLS Wiener filter [16]. The MSVs of the filtering errors by the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type and the centralized robust multi-sensor RLS Wiener filter are smaller than those of the centralized multi-sensor RLS Wiener filter. Also, the MSVs of the filtering errors by the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type and the centralized robust multi-sensor RLS Wiener filter are almost the same.

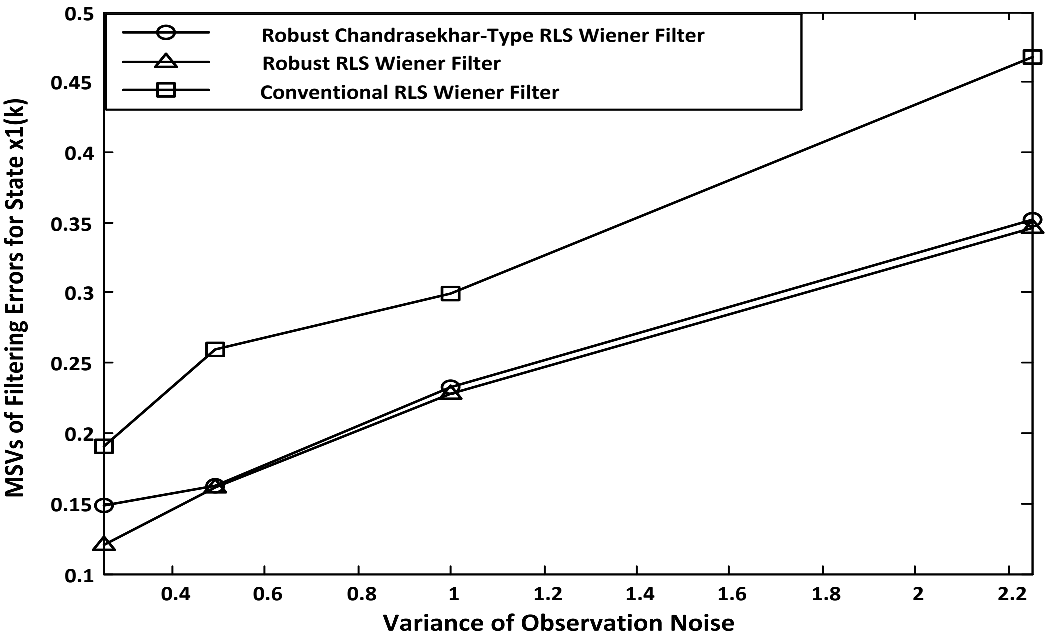

Fig. 3. MSVs of filtering errors of $x_1(k)$ vs. variance of white Gaussian observation noise by different types of centralized multi-sensor RLS Wiener filters.

Fig. 4 shows the MSVs of the filtering errors of $x_2(k)$ by the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1, the centralized robust multi-sensor RLS Wiener filter [15] and the centralized multi-sensor RLS Wiener filter [16]. The MSVs of the filtering errors by the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type and the centralized robust multi-sensor RLS Wiener filter are smaller than those of the centralized multi-sensor RLS Wiener filter. Also, the MSVs of the filtering errors by the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type and the centralized robust multi-sensor RLS Wiener filter are almost the same.

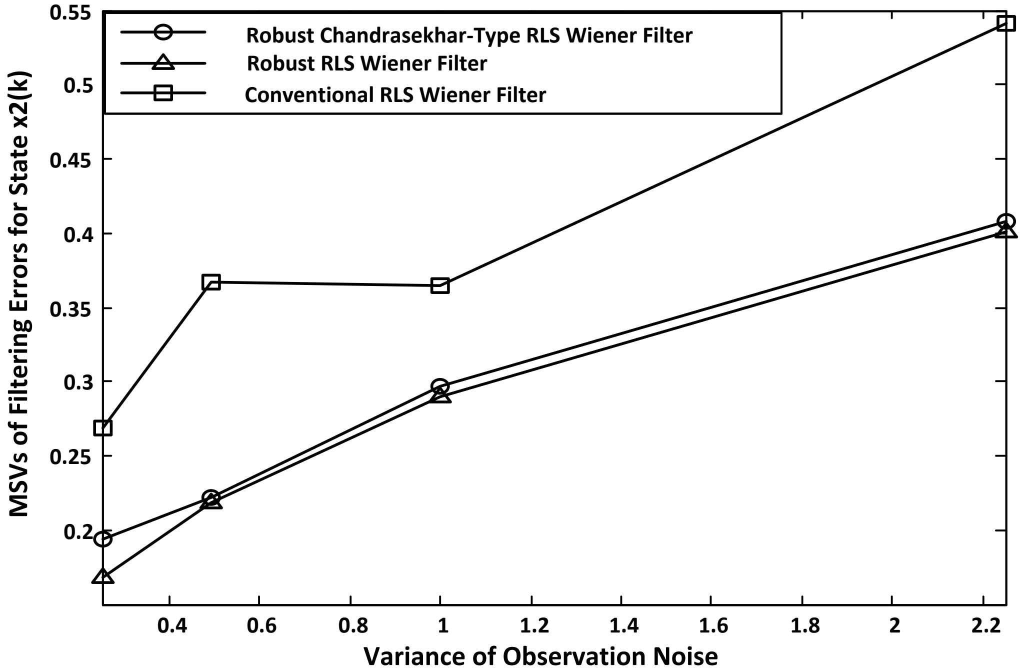

Fig. 4. Mean-square values of filtering errors of $x_2(k)$ by different types of centralized multi-sensor RLS Wiener filters.

7. CONCLUSIONS

This paper proposed, in Theorem 1, the centralized robust multi-sensor RLS Wiener filtering algorithm of Chandrasekhar type for the signal and the state in linear discrete-time wide-sense stationary stochastic systems with uncertain parameters.

In the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1, the filter gains $h(k,k)$ for $\hat{x}(k,k)$ and $\bar{h}(k,k)$ for $\hat{\hat{x}}(k,k)$ are recursively updated by Eqs. (14) and (16) without algebraic Riccati equations. From the numerical simulation example in section 6, the Chandrasekhar-type centralized robust multi-sensor RLS Wiener filter of Theorem 1 is stable in comparison with the centralized robust multi-sensor RLS Wiener filter for relatively small variance of the white Gaussian observation noise. The MSVs of the filtering errors by the centralized robust multi-sensor RLS Wiener filter are large for the white Gaussian observation noises $\mathrm{N}(0,0.2^2)$ and $\mathrm{N}(0,0.3^2)$, respectively. Interestingly, the centralized robust multi-sensor RLS Wiener filter of Chandrasekhar type is effective in estimation accuracy, for the relatively small variance of the observation noise, against the instability of the centralized robust multi-sensor RLS Wiener filter.

APPENDIX

Proof of Theorem1:

The optimal impulse response function $h(k,s)$ satisfies Eq. (11). Putting $k \to k-1$ and s $\to s-1$, we have

$$h(k-1,s-1)R = K_{x\overleftrightarrow{x}}(k-1,s-1)\overleftrightarrow{H}^T - \sum_{i=1}^{k-1} h(k-1,i)\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s-1)\overleftrightarrow{H}^T \quad \text{(A-1)}$$

Subtracting Eq. (A-1) from Eq. (11), we have

$$(h(k,s)-h(k-1,s-1)R = -h(k,1)\overleftrightarrow{H}\overleftrightarrow{K}(1,s)\overleftrightarrow{H}^T$$
$$-\sum_{i=2}^{k}\big(h(k,i)-h(k-1,i)\big)\,\overleftrightarrow{H}\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T - h(0,1)\overleftrightarrow{H}\overleftrightarrow{K}(1,s-1)\overleftrightarrow{H}^T \quad \text{(A-2)}$$
$$-h(k,1)\overleftrightarrow{H}\overleftrightarrow{K}(1,s)\overleftrightarrow{H}^T - \sum_{i=2}^{k}\big(h(k,i)-h(k-1,i)\big)\,\overleftrightarrow{H}\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T$$

Here, $h(0,1)$=0 is used. Introducing a function $J(\mathrm{k},\mathrm{s})$, which satisfies

$$J(k,s)R = \overleftrightarrow{H}\overleftrightarrow{K}(1,s)\overleftrightarrow{H}^T - \sum_{i=2}^{k} J(k,i)\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T, \quad \text{(A-3)}$$

we obtain

$$h(k,s)-h(k-1,s-1) = -h(k,1)J(k,s) \quad \text{(A-4)}$$

Let's introduce a function $\bar{h}(k,s)$, which satisfies

$$\bar{h}(k,s)R = \overleftrightarrow{K}(k,s)\overleftrightarrow{H}^T - \sum_{i=1}^{k}\bar{h}(k,i)\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T. \quad \text{(A-5)}$$

Putting $k \to k-1$ and pre-multiplying $\overleftrightarrow{H}\overleftrightarrow{\Phi}$ on both sides of Eq. (A-5), we get

$$\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,s)R = \overleftrightarrow{H}\overleftrightarrow{K}(k,s)\overleftrightarrow{H}^T - \overleftrightarrow{H}\overleftrightarrow{\Phi}\sum_{i=1}^{k-1}\bar{h}(k-1,i)\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T. \quad \text{(A-6)}$$

Putting s $\to k-s+1$ in Eq. (A-6), from the stationarity of $\overleftrightarrow{K}(k,s)$, we have

$$\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,k-s+1)R = \overleftrightarrow{H}\overleftrightarrow{K}(1,s)\overleftrightarrow{H}^T - \overleftrightarrow{H}\overleftrightarrow{\Phi}\sum_{i=2}^{k}\bar{h}(k-1,k-i+1)\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T. \quad \text{(A-7)}$$

From Eqs. (A-3) and (A-7), it follows that

$$J(k,s) = \overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,k-s+1), 2 \le s \le k. \quad \text{(A-8)}$$

From Eq. (A-4), it is clear that

$$h(k,k)-h(k-1,k-1) = -h(k,1)J(k,k) - h(k,1)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1) \quad \text{(A-9)}$$

Putting $k \to k-1$ in Eq. (A-5), we have

$$\bar{h}(k-1,s)R = \overleftrightarrow{K}(k-1,s)\overleftrightarrow{H}^T - \sum_{i=1}^{k-1}\bar{h}(k-1,i)\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T. \quad \text{(A-10)}$$

From Eqs. (A-5) and (A-10), we have

$$(\bar{h}(k,s) - \overleftrightarrow{\Phi}\bar{h}(k-1,s)\big)R =$$
$$-\bar{h}(k,k)\overleftrightarrow{H}\overleftrightarrow{K}(k,s)\overleftrightarrow{H}^T - \sum_{i=1}^{k-1}(\bar{h}(k,i)-\bar{h}(k-1,i))\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T. \quad \text{(A-11)}$$

From Eqs. (A-10) and (A-11), it is seen that

$$\bar{h}(k,s) - \overleftrightarrow{\Phi}\bar{h}(k-1,s) = -\bar{h}(k,k)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,s). \quad \text{(A-12)}$$

Hence, we obtain

$$\bar{h}(k,1) = \overleftrightarrow{\Phi}\bar{h}(k-1,1) - \bar{h}(k,k)\overleftrightarrow{H}\bar{h}(k-1,1). \quad \text{(A-13)}$$

Putting $k \to k-1$ and $s \to s-1$ in Eq. (A-5), we have

$$\bar{h}(k-1,s-1)R = \overleftrightarrow{K}(k-1,s-1)\overleftrightarrow{H}^T - \sum_{i=1}^{k-1}\bar{h}(k-1,i)\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s-1)\overleftrightarrow{H}^T. \quad \text{(A-14)}$$

Subtracting Eq. (A-14) from Eq. (A-5), we have

$$\Big(\bar{h}(k,s) - \bar{h}(k-1,s-1)\Big)R =$$
$$-\bar{h}(k,1)\overleftrightarrow{H}\overleftrightarrow{K}(1,s)\overleftrightarrow{H}^T - \sum_{i=2}^{k}(\bar{h}(k,i)-\bar{h}(k-1,i))\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T. \quad \text{(A-15)}$$

From Eqs. (A-3), (A-8), and (A-15), it follows that

$$\bar{h}(k,s) - \bar{h}(k-1,s-1) = -\bar{h}(k,1)J(k,s) = -\bar{h}(k,1)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,k-s+1). \quad \text{(A-16)}$$

Putting $s \to k$ in Eq. (A-16), we obtain

$$\bar{h}(k,k) = \bar{h}(k-1,k-1) - \bar{h}(k,1)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1). \quad \text{(A-17)}$$

The initial condition of the recursions for $\bar{h}(k,1)$ in Eq. (A-13) and $\bar{h}(k,k)$ in Eq. (A-17) is $\bar{h}(1,1)$. From Eq. (A-5), $\bar{h}(1,1)$ is given by:

$$\bar{h}(1,1) = \overleftrightarrow{K}(1,1)\overleftrightarrow{H}^T(R + \overleftrightarrow{H}\overleftrightarrow{K}(1,1)\overleftrightarrow{H}^T)^{-1}. \quad \text{(A-18)}$$

Putting $k \to k-1$ in Eq. (11), we have

$$h(k-1,s)R = K_{x\overleftrightarrow{x}}(k-1,s)\overleftrightarrow{H}^T - \sum_{i=1}^{k-1} h(k-1,i)\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T \quad \text{(A-19)}$$

From Eqs. (11) and (A-19), we have

$$\big(h(k,s) - \Phi h(k-1,s)\big)R = K_{x\overleftrightarrow{x}}(k,s)\overleftrightarrow{H}^T - \Phi K_{x\overleftrightarrow{x}}(k-1,s)\overleftrightarrow{H}^T - h(k,k)\overleftrightarrow{H}\overleftrightarrow{K}(k,s)\overleftrightarrow{H}^T - \sum_{i=1}^{k-1}(h(k,i) - h(k-1,i))\overleftrightarrow{H}\,\overleftrightarrow{K}(i,s)\overleftrightarrow{H}^T \quad \text{(A-20)}$$

From Eqs. (A-6) and (A-20), we get

$$h(k,s) = \Phi h(k-1,s) - h(k,k)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,s) \quad \text{(A-21)}$$

Putting $s \to 1$, we obtain

$$h(k,1) = \Phi h(k-1,1) - h(k,k)\overleftrightarrow{H}\overleftrightarrow{\Phi}\bar{h}(k-1,1) \quad \text{(A-22)}$$

The initial condition of the recursions for $h(k,k)$ in Eq. (A-9) and $h(k,1)$ in Eq. (A-22) is $h(1,1)$. From Eq. (11) $h(1,1)$ is given by

$$h(1,1) = K_{x\overleftrightarrow{x}}(1,1)\overleftrightarrow{H}^T(R + \overleftrightarrow{H}\overleftrightarrow{K}(1,1)\overleftrightarrow{H}^T)^{-1}. \quad \text{(A-23)}$$

From Eqs. (A-9) and (A-22), we obtain Eq. (14). From Eqs. (A-13) and (A-17), we obtain Eq. (16).

The filtering estimate $\hat{x}(k,k)$ of $x(k)$ is given by Eq. (10). Using Eq. (A-21), we rewrite Eq. (10) as follows.

$$\begin{aligned}\hat{x}(k,k) &= h(k,k)\breve{y}(k) + \sum_{i=1}^{k-1} h(k,i)\,\breve{y}(i)\\ &= h(k,k)\breve{y}(k) + \Phi\sum_{i=1}^{k-1} h(k-1,i)\,\breve{y}(i) - h(k,k)\overleftrightarrow{H}\overleftrightarrow{\Phi}\sum_{i=1}^{k-1}\bar{h}(k-1,i)\,\breve{y}(i)\\ &= \Phi\hat{x}(k-1,k-1) + h(k,k)(\breve{y}(k) - \overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{x}(k-1,k-1))\end{aligned} \quad \text{(A-24)}$$

Initial condition on the recursive equation for the filtering estimate $\hat{x}(k,k)$ at $k = 0$ is $\hat{x}(0,0) = 0$ from Eq. (10). By the way, $\bar{h}(k,s)$ in Eq. (A-5) is used to calculate the filtering estimate $\hat{\overleftrightarrow{x}}(k,k)$ of $\overleftrightarrow{x}(k)$ as

$$\hat{\overleftrightarrow{x}}(k,k) = \sum_{i=1}^{k}\bar{h}(k,i)\,\breve{y}(i) \quad \text{(A-25)}$$

Substituting Eq. (A-12) into Eq. (A-25), we obtain

$$\begin{aligned}\hat{\overleftrightarrow{x}}(k,k) &= \bar{h}(k,k)\breve{y}(k) + \sum_{i=1}^{k-1}\bar{h}(k,i)\,\breve{y}(i)\\ &= \bar{h}(k,k)\breve{y}(k) + \overleftrightarrow{\Phi}\sum_{i=1}^{k-1}\bar{h}(k-1,i)\,\breve{y}(i) - \bar{h}(k,k)\overleftrightarrow{H}\overleftrightarrow{\Phi}\sum_{i=1}^{k-1}\bar{h}(k-1,i)\,\breve{y}(i)\\ &= \Phi\hat{\overleftrightarrow{x}}(k-1,k-1) + \bar{h}(k,k)(\breve{y}(k) - \overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(k-1,k-1)).\end{aligned} \quad \text{(A-26)}$$

Initial condition on the recursive equation for the filtering estimate $\hat{\overleftrightarrow{x}}(k,k)$ at $k = 0$ is $\hat{\overleftrightarrow{x}}(0,0) = 0$ from Eq. (A-25).

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